Applications of Conformal Mapping in Fluid Mechanics: A Review

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ABSTRACT

Conformal mapping transforms the curves sustaining their angle is a great tool in complex analysis to map the domain (of non interest) to sought after domain. In this article we relieved the applications and importance of conformal mapping in the fluid mechanics, in particular in an irrotational flow of incompressible flow i.e. the flow of an ideal fluid in the complex plane.

Introduction

There are infinitely many problems in fluid mechanics that are difficult to solve in their original form in the given domain. Conformal mapping easily translates an equation and a domain from its original form into another, after some mathematical manipulations we get the solution and the solution can be sent back into the original form. For solving such problems in engineering and physics that are expressible as a function of a complex variable, but that show clumsy geometry, conformal mapping is vital. By choosing suitable function, the inconvenient geometry can be transformed into a much more suitable one. For example, one may be interested of calculating the velocity potential of flow of an ideal fluid in an elliptic pipe that make certain angle. This problem is quite awkward to solve in its original form. However, by using a simple conformal mapping, the inconvenient curved angle is mapped to one of precisely $\pi$, it means that the two curved boundaries are transformed to a straight line and in the new region, the problem of calculating the velocity potential is easy.

Preliminaries

Conformal Mapping [10-16]

Let $w = f(z)$ be a complex mapping defined in a domain $D$ and let $z_0$ be a point in $D$. Then we say that $w = f(z)$ is conformal at $z_0$ if for every pair of smooth oriented curves $C_1$ and $C_2$ in $D$ intersecting at $z_0$ the angle between $C_1$ and $C_2$ at $z_0$ is equal to the angle between the image curves $C_1'$ and $C_2'$ at $f(z_0)$ in both magnitude and sense.

Theorem

If $f$ is an analytic function in a domain $D$ containing $z_0$, and if $f'(z_0) \neq 0$, then $w = f(z)$ is a conformal mapping at $z_0$.

Theorem

Let $f(z) = u(x,y) + iv(x,y)$ be an analytic mapping of a domain $D$ in the $z$-plane onto a domain $D'$ in the $w$-plane. If the function $\Phi(u,v)$ is harmonic in $D'$, then the function $\phi(x,y) = \Phi(u(x,y), v(x,y))$ is harmonic in $D$.

Discussion

There are lot of features of conformal mapping that can be used for various practical applications in distinct field although the essence remains the same. It preserves the shape and angle locally and maps of harmonic functions to the harmonic functions. These properties of conformal mapping make it beneficial in complex conditions, specifically complex velocity potential problems for general systems. However, the conformal mapping approach is restricted to problems that are two dimensional and to problems involving high degrees of symmetry. It is sometimes
impossible to apply this technique when there is no symmetry. Also, this technique is applicable only for irrotational flow of an ideal fluid, as it depends on the harmonicity of the velocity potential which comes from the Laplace equation, an equation of continuity of an incompressible fluid. Furthermore, it gives the complex velocity potential of flow of an ideal fluid in the region that can be either mapped onto horizontal strip or vertical strip under the one-to-one, onto, conformal mapping. Following figure shows that the conformal mapping of harmonic function to harmonic function from one domain to another domain.

![Conformal Mapping](image)

**Conclusion**

Conformal mapping helps in producing flows of an ideal fluid when the flow is two-dimensional with no trade off in the laws of fluid mechanics. However, it does not work for the practical three-dimensional flows found in nature.

**References**